3. R. S. Mikhal'chenko, N. P. Pershin, and L. V. Klipach, Hydrodynamics and Heat Transfer in Cryogenic Systems [in Russian], Kiev (1977), pp. 79-85.
4. N. P. Pershin and R. S. Mikhal'chenko, Heat Transfer at Low Temperatures [in Russian], Kiev (1979), pp. 56-68.

GENERALIZED EQUATION FOR KINETICS OF CONVECTIVE DRYING OF MOIST MATERIALS

P. S. Kuts, V. Ya. Sklyar, and A. I. Ol'shanskii

UDC 001.57:685.31.001.5

A generalized equation describing the drying process in continuous-acting convective dryers is derived and analyzed. Results of a numerical solution are compared with experimental data.

An equation was derived in [1] which established the dependence of evaporated moisture output from a convective dryer upon drying time, kinetic characteristics of the process, properties of the material being dried, and flow rate and temperature of the heat-exchange agent. To generalize that study it will be desirable to derive the fundamental equation in dimensionless form, allowing a significant expansion in its practical application range.

The thermal balance equation for a continuous action convective dryer has the form:

$$[\alpha_{cr}(T_h - T_w) + \bar{\alpha}(T_h - T_s)]F = rm_0 N + rm_0 \frac{d\bar{u}}{d\tau} + (c_0 m_0 + c_m m_m^{II}) \frac{dT_{mt}}{d\tau}. \quad (1)$$

Following [1], Eq. (1) can be represented as

$$[St_{cr}(T_h - T_w) + \bar{St}(T_h - T_s)]F = \frac{1}{c_p v\rho}\left[rm_0 N + rm_0 \frac{d\bar{u}}{d\tau} + (c_0 m_0 + c_m m_m^{II}) \frac{dT_{mt}}{d\tau}\right], \quad (2)$$

where the Stanton numbers St_{cr} and $\bar{St}$ are dimensionless characteristics of the intensity of heat transport in the first and second stages of drying.

The product of two generalized parameters, the relative drying rate N* and the Rebinder number Rb, can be written in the form

$$N^*Rb = \left(\frac{1}{N}\frac{d\bar{u}}{d\tau}\right)\left(\frac{dT_{mt}}{d\bar{u}}\frac{c_{mt}}{r}\right), \quad (3)$$

while the drying rate in the second stage [2] is given by:

$$\left|\frac{d\bar{u}}{d\tau}\right| = \varkappa N(\bar{u} - u_e). \quad (4)$$

It follows from Eqs. (3), (4) that

$$\frac{dT_{mt}}{d\tau} = \frac{r}{c}\varkappa N(\bar{u} - u_e)\,Rb. \quad (5)$$

With consideration of this last relationship, Eq. (2) takes on the form

$$[St_{cr}(T_h - T_w) + \bar{St}(T_h - T_s)]F = \frac{rN}{c_p v\rho}\left[m_0 + m_0\varkappa(\bar{u} - u_e) + (c_0 m_0 + c_m m_m^{II})\frac{\varkappa}{c_{mt}}(\bar{u} - u_e)\,Rb\right]. \quad (6)$$

We will divide both sides of Eq. (6) by the expression $St_{cr}(T_h - T_w) = (T_1 - T_2)f/F$ [3]. With consideration of the expressions [4]

A. V. Lykov Heat and Mass Transfer Institute, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 1, pp. 90-96, July, 1987. Original article submitted April 29, 1986.

 0022-0841/87/5301-0812$12.50

$$\frac{\overline{\mathrm{St}}}{\mathrm{St_{cr}}} \equiv \frac{\overline{\mathrm{Nu}}}{\mathrm{Nu_{cr}}} = (1+\mathrm{Rb})\,N^{*0.57}; \tag{7}$$

$$\frac{T_h - T_s}{T_h - T_w} = \Delta T^* = N^{*0.43} \tag{8}$$

Eq. (6) can be written in the form

$$1+(1+\mathrm{Rb})\,N^* = \frac{rN}{c_p v \rho f (T_1 - T_2)}\left[m_0 + m_0 \varkappa (\bar{u} - u_e) + (c_0 m_0 + c_m m_m^{II}) \frac{\varkappa}{c_{mt}} (\bar{u} - u_e)\,\mathrm{Rb}\right] \tag{9}$$

or

$$1+(1+\mathrm{Rb})\,N^* = \frac{rNm_0}{c_p v \rho f (T_1 - T_2)}\left[1 + N^* + \frac{c_0 + c_m \frac{m_m^{II}}{m_0}}{c_{mt}} N^* \mathrm{Rb}\right]. \tag{10}$$

We will now consider the ratio of two quantities — the Fourier number Fo and the Reynolds number Re:

$$\frac{\mathrm{Fo}}{\mathrm{Re}} = \frac{\lambda\tau}{c_p \rho b^2} : \frac{vb}{\nu}. \tag{11}$$

With consideration of this

$$\frac{1}{c_p v \rho} = \frac{\mathrm{Fo}}{\mathrm{Re}} \frac{b^3}{\lambda \tau \nu}. \tag{12}$$

In accordance with [2] the Kirpichev number Ki_0 for the constant rate drying period is defined by the expression:

$$\mathrm{Ki}_0 = \frac{q^I R_V}{\lambda_{mt} T_h}, \tag{13}$$

with the density of the thermal flux supplied to the material being

$$q^I = r \rho_0 R_V N. \tag{14}$$

We assume that the geometric dimensions of the material do not change during drying. Then the quantity R_V represents the ratio of the volume of moist material V to its surface 2F (the material is drafted by heat-exchange agent from both sides):

$$R_V = \frac{V}{2F} = \frac{Ab\delta}{2Ab} = \frac{\delta}{2}, \tag{15}$$

where A and δ are the length and thickness of the material being dried in m.

With consideration of Eq. (15), the expression for the Kirpichev number takes on the form

$$\mathrm{Ki}_0 = \frac{r \rho_0 R_V^2 N}{\lambda_{mt} T_h},$$

where

$$N = \frac{4 \mathrm{Ki}_0 \lambda_{mt} T_h}{r \rho_0 \delta^2}. \tag{16}$$

The Kossovich number Ko, characterizing the relationship between the amounts of heat expended in evaporating moisture and heating the moist material can be written as:

$$\mathrm{Ko} = \frac{r\bar{u}}{c_{mt} T_h}. \tag{17}$$

The current output of the dryer is

$$\bar{w} = \frac{\bar{m}_m}{\tau}, \tag{18}$$

where $\bar{m}_m$ is the quantity of moisture in the material in kg. This quantity can be represented as

$$\bar{m}_m = m_0\bar{u}. \tag{19}$$

Since $a_{mt} = \lambda_{mt}/\rho_0 c_{mt}$ (the thermal diffusivity coefficient of the material) and $Pr = \nu/a$, with consideration of Eqs. (12), (16), (17), (18), (19), Eq. (10) can be written in the form

$$1+(1+Rb)N^* = \frac{FoKi_0}{RePrKo}\,\frac{a_{mt}}{a}\,\frac{4r\bar{w}b^3}{\lambda\delta^2(T_1-T_2)f}\left[1+N^*+\frac{c_0+c_m\frac{m_m^{II}}{m_0}}{c_{mt}}N^*Rb\right]. \tag{20}$$

We will introduce the following notation: $a_{mt}/a = Lu^*$, a number characterizing the ratio of heat propagation within the material to its propagation in the boundary layer on the surface of the material; $r\bar{w}/\lambda\delta(T_1 - T_2) = \Pi^*$, a dimensionless complex describing moisture loss in the material during drying; $4b^3/\delta f = l^*$, a dimensionless complex considering the relationship between geometric parameters of the dryer and material being dried;

$$n = \frac{m_m^I}{\bar{m}_m}; \quad 1-n = \frac{m_m^{II}}{\bar{m}_m}. \tag{21}$$

The quantity n represents the ratio of the mass of moisture evaporated in the first period to the entire amount of moisture evaporated from the material, and is dependent upon the form of the drying curve [1].

On the basis of Eqs. (19) and (21)

$$\frac{m_m^{II}}{m_0} = (1-n)u_0. \tag{22}$$

The heat capacity of the moist material is defined by the expression $c_{mt} = c_0 + c_m\bar{u}$. Following [2], we denote: $(c_0 = c_m u_0(1 - n))/(c_0 + c_m\bar{u}) = K_h^*$, a modified dimensionless complex characterizing the heat capacity of the moist material.

With consideration of the notation defined above Eq. (20) takes on the form

$$RePrKo\,[1+(1+Rb)N^*] = FoKi_0Lu^*\Pi^*l^*[1+(1+K_h^*Rb)N^*]. \tag{23}$$

The generalized equation (23) describes the process of drying of moist materials in continuous action convective dryers, relating the basic kinetic characteristics of the process to the properties of the material being dried and the heating agent as well as the geometric characteristics of the dryer and material and the temperature and hydrodynamic conditions under which the process occurs. It can be used for modeling drying processes occurring in convective type industrial dryers.

The case of greatest interest for practical engineering calculations is that of determining dryer output when the material is being dried to an equilibrium moisture content ($\bar{u} = u_e$, $N^* = 0$ and $Rb = Rb_{\bar{u}=u_e} = const$). In this case, Eq. (23) can be written as

$$RePrKo = FoKi_0Lu^*\Pi^*l^*, \tag{24}$$

whence

$$\Pi^* = \frac{RePrKo}{FoKi_0Lu^*l^*}. \tag{25}$$

Equation (25) relates (in dimensionless form) the output of the convective dryer to the basic characteristics controlling the drying process.

We will consider the effect of these quantities upon the parameter Π^*, with the fundamental quantity varied being the output of the dryer w. The values of the other parameters varied were chosen in accordance with their real ranges of variation for operation of "Éliteks" type convective dryers (drying of fabric with two-sided nozzle draft). Table 1 presents ranges for all dimensionless parameters appearing in Eq. (25).

Results of a numerical study are presented in Figs. 1-3. Analysis of the graphs reveals that all the dependences are hyperbolic in character with the quantitative effect of the varied parameters on the magnitude of the dimensionless complex varying, the greatest effect be-

TABLE 1. Range of Variations of Dimensionless Variables of Eq. (25)

Quantity varied	Constant values
$(RePr)\cdot 10^{-6}=2-5$	$Ko=6{,}4$; $Lu^*=6{,}8$; $Ki_0\cdot 10^3=1{,}7$; $l^*\cdot 10^{-3}=5$
$Ko=2-10$	$(RePr)\cdot 10^{-6}=2{,}6$; $Lu^*=6{,}8$; $Ki_0\cdot 10^3=1{,}7$; $l^*\cdot 10^{-3}=5$
$Lu^*=2-10$	$(RePr)\cdot 10^{-6}=2{,}6$; $Ko=6{,}4$; $Ki_0\cdot 10^3=1{,}7$; $l^*\cdot 10^{-3}=5$
$Ki_0\cdot 10^3=1-50$	$(RePr)\cdot 10^{-6}=2{,}6$; $Ko=6{,}4$; $Lu^*=6{,}8$; $l^*\cdot 10^{-3}=5$
$l^*\cdot 10^{-3}=2-10$	$(RePr)\cdot 10^{-6}=2{,}6$; $Ko=6{,}4$; $Ki_0\cdot 10^3=1{,}7$; $Lu^*=6{,}8$

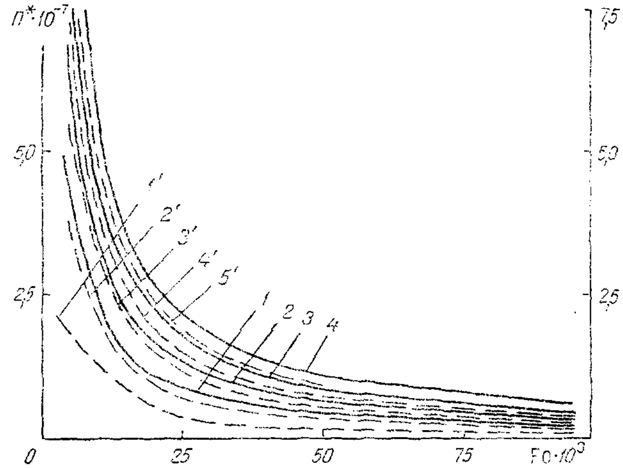

Fig. 1. Dimensionless complex П* vs Fourier number Fo and Reynolds number Re (solid curves) and Kossovich number Ko (dashes): 1) RePr = 2•10⁶: 1') Ko = 2; 2) RePr = 3•10⁶; 2') Ko = 4; 3) RePr = 4•10⁶; 3') Ko = 6; 4) RePr = 5•10⁶; 4') Ko = 8; 5') Ko = 10.

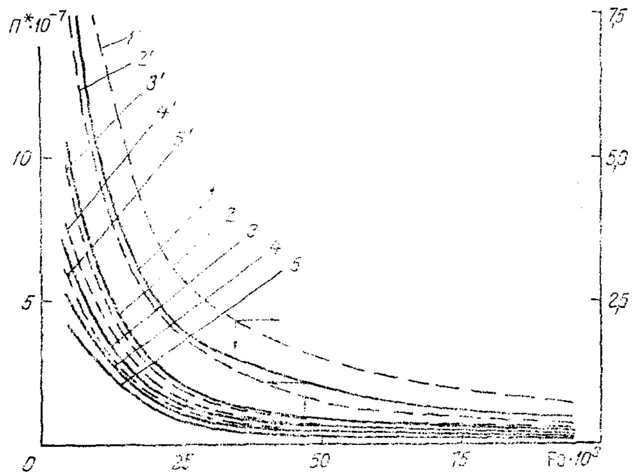

Fig. 2. Dimensionless complex П* vs Fourier number Fo, number Lu* (solid curves) and dimensionless complex l^* (dashes): 1) Lu* = 2; 1') $l^* = 2\cdot 10^3$; 2) Lu* = 4; 2') $l^* = 4\cdot 10^3$; 3) Lu* = 6; 3') $l^* = 6\cdot 10^3$; 4) Lu* = 8; 4') $l^* = 8\cdot 10^3$; 5) Lu* = 10; 5') $l^* = 10\cdot 10^3$.

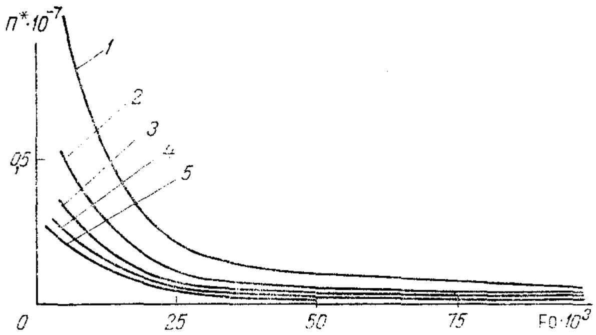

Fig. 3. Dimensionless complex Π^* vs Fourier number Fo and Kirpichev number Ki_0: 1) $Ki_0 = 10^{-2}$; 2) $2 \cdot 10^{-2}$; 3) $3 \cdot 10^{-2}$; 4) $4 \cdot 10^{-2}$; 5) $5 \cdot 10^{-2}$.

TABLE 2. Comparison of Numerical Solution with Experiment

$Fo \cdot 10^3$	$\Pi^* \cdot 10^{-7}$	
	numerical solution of (25)	expt.
28,0	1,04	1,02
21,5	1,35	1,32
22,0	1,38	1,31

ing found at values up to $Fo = 50 \cdot 10^{-3}$. At $Fo > 50 \cdot 10^{-3}$ this effect decreases greatly, corresponding to the drying process in the moisture content range $\bar{u} \to u_e$.

With increase in the Reynolds number Re and Kossovich number Ko (the Prandtl number Pr remains practically constant) the value of Π^* increases (Fig. 1), since in this case external heat exchange is intensified and the major portion of the heat supplied is expanded in evaporation of moisture from the material.

With increase in the number Lu^* and the dimensionless complex l^* the quantity of moisture evaporated decreases (Fig. 2), since the intensity of heat propagation in the boundary layer at the material surface, the material thickness, and the mass of moisture therein decrease (for f, b = const).

With increase in the Kirpichev number Ki_0 (Fig. 3) the quantity of moisture evaporated also decreases, since with reduction in temperature of the heat-exchange agent T_h the drying rate in the first period also decreases (for Re = const).

Table 2 presents values of Π^* obtained in experiments on drying of fabrics in "Éliteks"-type convective dryers. Also shown for comparison are values of the same complex calculated with Eq. (25).

It is evident from the table that satisfactory agreement exists between the experimental results and solution of the proposed generalized equation.

NOTATION

α_{cr}, $\bar{\alpha}$, heat liberation coefficients in first and second stages, $W/(m^2 \cdot K)$: T_h, T_w, T_s, temperatures of heat-transfer agent, wet bulb thermometer, and material surface, °K; F, material surface area, m^2; τ, drying time, sec; m_0, m_m^I, m_m^{II}, mass of dry material and masses of moisture evaporated during first and second stages, kg; N, $du/d\tau$, drying rates in first and second stages, sec^{-1}; $dT_{mt}/d\tau$, rate of change of material temperature, K/sec; r, latent heat of evaporation, J/kg; c_0, c_{mt}, c_m, specific heats of dry material, moist material, and moisture, J/kg·deg; c_p, v, ρ, isobaric specific heat of heating agent, J/(kg·K), velocity relative to nozzle output section, m/sec, and density, kg/m^3; f, total area of nozzle output section, m^2; b, width of material (controlling dimension), m; T_1, T_2, heating agent temperature at input and output of drying chamber, °K; $\varkappa$, relative drying coefficient; $\bar{u}$, u_e, current and equilibrium moisture content of material; λ, ν, a, thermal conductivity, W/(m•K), kinematic viscosity, m^2/sec, and thermal diffusivity, m^2/sec, of heating agent; λ_{mt}, ρ_0, thermal conductivity W/(m•K) and density kg/m^3 of material.

LITERATURE CITED

1. P. S. Kuts, V. Ya. Sklyar, and A. I. Ol'shanskii, Inzh.-Fiz. Zh., 51, No. 1, 99-104 (1986).
2. A. V. Lykov, Theory of Drying [in Russian], Moscow (1968).
3. A. A. Gukhman, Application of Similarity Theory to Study of Heat Mass Transport Processes [in Russian], Moscow (1967).
4. A. V. Lykov, P. S. Kuts, and A. I. Ol'shanskii, Inzh.-Fiz. Zh., 23, No. 3, 401-406 (1972).

METHODS FOR PERFORMING ENGINEERING CALCULATIONS OF THE PROCESS OF VACUUM DRYING OF HEAVY-DUTY CAPACITORS

N. A. Prudnikov and N. A. Gudko

UDC (621.319.4:621.315.614.6).002.2

A method is proposed for calculating the instantaneous average values of the temperature and moisture content of the insulation of capacitors as a function of the parameters of the drying process.

Heavy-duty capacitors, as objects of heat treatment, are complicated structures. The process of their thermovacuum drying is also quite complicated from the physical viewpoint. All these characteristics are responsible for the great complexity of the physical and mathematical modeling of these processes and the fact that there do not exist reliable engineering methods for calculating them. At the same time, such methods are required not only by designers of such electrothermal equipment, but also by manufacturers.

We shall first study the thermophysical model of a capacitor (Fig. 1). The presence of a foil interlayer 1 substantially affects the thermal conductivity of the system as a whole. We assume that at the lower boundary we have the most general case — boundary conditions of the second kind, and in addition the heat flux is time dependent. To a first approximation the heat expended on the evaporation of moisture in the insulation can be neglected. Since the thermal conductivity of the system along the X axis is several orders of magnitude higher than the thermal conductivity in the transverse direction we shall study the one-dimensional problem. In so doing we assume that the temperature gradient in the transverse direction within one layer of paper will be vanishingly small. Then, the energy expended on heating the paper adjacent to the foil can be taken into account as the draining of heat from the foil, and heat conduction along the foil only can be studied. This approach is fully justified, since heat transfer by conduction along the paper is several orders of magnitude weaker than along the foil

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{\partial^2\theta}{\partial X^2}, \quad \theta(X,\ 0) = 1, \quad \frac{\partial\theta(0,\ \mathrm{Fo})}{\partial X} = -\mathrm{Sk}(\theta_w^4 - \theta^4), \quad \frac{\partial\theta(1,\ \mathrm{Fo})}{\partial X} = 0. \tag{1}$$

Here $\theta = T/T_0$; $X = x/L_0$; $\mathrm{Fo} = a\tau/L_0^2 b$; $\mathrm{Sk} = \varepsilon\sigma_0 T_0^3/\lambda$; $b = (c_1\rho_1\delta_1/c_2\rho_2\delta_2 + 1)$ is a coefficient that takes into account the additional heat lost to heating the paper or film adjacent to the foil.

Equation (1) was represented in an implicit difference form, which was highly stable, and was solved by the straight iteration method.

The calculations show that the difference of the temperatures at the outer and inner layers does not exceed 1°C. These results are confirmed by the experimental data of [1], where large temperature gradients also were not recorded, which gives a basis for neglecting in further calculations the internal heat conduction in heavy-duty capacitors.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 1, pp. 96-101, July, 1987. Original article submitted February 6, 1986.

0022-0841/87/5301-0817$12.50